Allocation of Time, Human Capital and Endogenous Growth

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1. - Introduction

In the second half of the 80s widespread attention has again been paid to growth theory, after the "wave" of business cycle studies.

This renewed interest originates from the pioneering work of Romer [9] and from the following papers by Rebelo [8], Lucas [6] and Barro [3].

This new line of research succeeds in explaining some stylized facts that a neoclassical growth theory (1) model could not deal with. Basically these stylized facts are: 1) long-run growth of per capita values; 2) non convergence among economies having the same structural parameters (2); 3) relevance of the savings rate in affecting the rate of growth of the economy.

With regard to the first stylized fact, a neoclassical model predicts zero rate of growth of per capita income, capital and consumption, due to decreasing returns in the reproducible factors; therefore in steady state the rate of growth of the economy is exogenously determined by the rate of growth of population. The observed in-

(1) Thrughout the paper we will refer to neoclassical model, meaning the Ramsey-Cass-Koopmans model.

Advise: the numbers in square brackets refer to the Bibliography in the appendix.

(2) By structural parameters we mean: 1) intertemporal elasticity of substitution; 2) discount rate; 3) labor and capital share.

creases in per capita values are explained by exogenous technical progress.

With regard to the second stylized fact, the neoclassical model says that if a rich and a poor (3) economy have the same structural parameters, they will converge to the same steady state, the poorer country growing faster due to the higher marginal productivity of its capital.

The last fact is strictly connected to the first one; as the rate of growth of the economy is determined solely by that of population, the savings rate has no role in affecting it. A one-time increase in the savings rate will simply augment per capita values; only in the transition to the new steady state will the rate of growth of the economy be temporarily higher, while in the new steady state it will again be determined by the exogenous demographic growth.

These three predictions are clearly counterfactual and furthermore strongly limit the role of economic policy in determining the growth path of an economy.

The crucial hypothesis needed to obtain self-sustained ("endogenous") growth, i.e. per capita growth explained by the model itself, and to account for the stylized facts mentioned above, is that of constant returns on the reproducible factors (4).

Given this, a great effort of investigation has been devoted to offering plausible explanations for nondecreasing returns to capital, meant as the complex of reproducible factors.

Among the researchers who have contributed most to this branch of the literature, Rebelo offers no explanation, Romer considers technological externalities and Barro public expenditure externalities; finally Lucas turns labor as well into a reproducible factor by allowing for human capital accumulation.

All these different explanations ensure constant returns to scale (CRS) to capital considered in a broad sense; all models argue that such a technology feature constitutes a sufficient condition for endogenous growth.

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(3) Given two economies, the one with a lower stock of capital is poor.

(4) As long as the production function includes also non-reproducible factors, it exhibits overall increasing returns to scale. For problems concerning the existence of a set of prices supporting a general competitive equilibrium see the survey of SALA-I-MARTIN [10].

On the other hand, we have observed that all these models share not only such a description of technology, but also the hypothesis that a fixed proportion of the available time of economic agents is in some way devoted to accumulating some kind of capital. In all models but Lucas', time devoted to accumulation is not under choice. Only Lucas allows for an optimal allocation of time between studying and working, but these two activities in any case accumulate reproducible factors, human and physical capital respectively. Hence what matters is the sum of studying and working time, which is actually a fixed proportion of total time. Whatever the individual decides to do, he always spends the same hours of his day accumulating!

In this paper we simply mean to demonstrate that a technology with CRS on reproducible factors is not by itself a sufficient condition for endogenous growth. We want to show that it needs to be accompanied by the afore-mentioned condition on the allocation of time, taken for granted in all models, as far as we know. In other words, if we allow for an endogenous choice of the total time devoted to accumulation, it happens that, despite the CRS technology, we come back to exogenous neoclassical growth.

In order to prove this assertion we take into account the individual choice concerning the optimal allocation of time at any instant among leisure, studying and working. This aspect represents the key issue we addressed in our previous research on human capital accumulation and saving behavior in the life-cycle. In two papers we formulated two different optimization problems, a simple two-period stochastic model (Baldassarri, De Santis, Moscarini, Piga [1]) and a deterministic dynamic continuous time model (Baldassarri *et* Al. [2]). The explicit consideration of the optimal allocation of time allowed us to endogenously determine labor supply and income, human capital accumulation, career evolution, consumption and savings.

The natural extension of this framework to optimal growth theory led us to the endogenous growth literature and similarly permitted us to highlight some of its unexplored features. In particular, the possibility of choosing leisure brings us to the argument of this paper, the interpretation of which is straightforward; if the individual is not compelled to accumulate at a constant rate reproducible factors with CRS, endogenous growth is no longer a granted result.

Endogenous labor supply through intertemporal substitution with leisure has received considerable attention in business cycle literature, where it has been considered a major source of fluctuations in output. Yet, its role has been widely neglected in long-run growth theory.

We shall prove our assertion for two classes of models (5): endogenous growth with human capital accumulation (Lucas [6], section 2) and endogenous growth with externalities (Romer [9], section 3) (6). In section 4 we shall explain the mathematical conclusions reached in the other sections and section 5 offers a conclusion.

2. - Lucas' Model with Leisure

We now consider Lucas' model [6] of endogenous growth the "engine" of which is human capital accumulation. The social planner maximises:

$$\max_{\{c_t, l_t, u_t\}_0^\infty} \int_0^\infty \frac{N_t\,[c_t^{(1-\sigma)} + l_t^{(1-\sigma)}]}{1-\sigma} e^{-\pi t}\, dt \tag{1}$$

s.t.

$$\frac{dH_t}{dt} = \delta\, H_t\,(1 - l_t - u_t) \tag{2}$$

$$\frac{dK_t}{dt} = AK_t^\beta\,(u_t\, H_t)^{(1-\beta)}\, H_t^\gamma - N_t\, c_t \tag{3}$$

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(5) We omit a similar treatment to the *AK* Rebelo model, although it represents a common benchmark and all other models are simple microfoundations of its CRS technology. Actually, the introduction of leisure, and consequently of endogenous working time, obviously requires that labor be explicitly considered. In the *AK* model, however, all factors are reproducible and exhibit overall CRS; hence labor has also to be taken as a reproducible factor (which is stated by Rebelo himself), i.e. as human capital, and this can be simply accounted for in Lucas' framework, which seems to us a more general reference.

(6) The analysis carried out on the Romer's framework is valid for any externality based endogenous growth model, including Barro [3].

Small letters represent values in per capita terms, the utility function is a standard *CRRA*, σ is the inverse of the intertemporal elasticity of substitution, equal for the two arguments, π is the subjective rate of discount, K_t is physical capital, N_t is the size of population, c_t is per capita consumption, l_t and u_t are leisure and working time respectively, both expressed as a share of the unit of time dt, H_t is aggregate human capital (7), δ is the maximum rate of accumulation of human capital, γ is the externalities parameter, β and $(1 - \beta)$ are the capital and labor share respectively and A is a constant parameter.

In order to solve this problem, set the following current Hamiltonian value:

$$R = \frac{N_t\,[c_t^{(1-\sigma)} + l_t^{(1-\sigma)}]}{1-\sigma} +$$

$$+ \theta_{1t}\,[AK_t^{\beta}(u_tH_t)^{(1-\beta)}H_t^{\gamma} - N_tc_t] + \theta_{2t}\,\delta\,(1 - l_t - u_t)\,H_t$$

The necessary conditions for an interior maximum are:

$$c_t^{-\sigma} = \theta_{1t} \tag{4}$$

$$N_t l_t^{-\sigma} = \theta_{2t}\,\delta\,H_t \tag{5}$$

$$\theta_{1t}\,A\,(1-\beta)\,K_t^{\beta}\,H_t^{(1-\beta)}\,u_t^{-\beta}\,H_t^{\gamma} = \theta_{2t}\,\delta\,H_t \tag{6}$$

$$\frac{d\theta_{1t}}{dt} = \pi\theta_{1t} - \theta_{1t}\,A\beta K_t^{(\beta-1)}\,(u_tH_t)^{(1-\beta)}\,H_t^{\gamma} \tag{7}$$

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(7) With regard to Lucas' model, here we have introduced a slight modification. As a matter of fact in Lucas' model human capital is considered in per capita terms, while the other reproducible factor, physical capital, is in aggregate terms. This asymmetry is reflected in the first order condition that equalizes the marginal benefits from studying and working; in Lucas' model such a condition sets equal the aggregate gain from the last instant of working to the individual gain from the last instant of studying. Allowing for leisure, if H_t were in per capita terms in capita, the necessary condition on leisure would equalize the aggregate marginal benefit from leisure to the individual marginal benefit from studying eq. *(5)*. In any case our results of zero per capita growth rates would not be affected, if we worked with per capita human capital.

$$(8) \qquad \frac{d\theta_{2t}}{dt} = \pi\theta_{2t} - \theta_{1t} A (1 - \beta + \gamma) K_t^{\beta} H_t^{(\gamma-\beta)} u_t^{(1-\beta)} - \theta_{2t} \delta (1 - l_t - u_t)$$

$$(2) \qquad \frac{dH_t}{dt} = \delta H_t (1 - l_t - u_t)$$

$$(3) \qquad \frac{dK_t}{dt} = A K_t^{\beta} (u_t H_t)^{(1-\beta)} H_t^{\gamma} - N_t c_t$$

$$(9) \qquad \lim_{t \to \infty} K_t \theta_{1t} e^{-\pi t} = 0$$

$$(10) \qquad \lim_{t \to \infty} H_t \theta_{2t} e^{-\pi t} = 0$$

Define λ as the rate of growth of population, χ as the rate of growth of per capita consumption, ξ as the rate of growth of aggregate capital and γ as the rate of growth of aggregate human capital.

Equation *(4)* says that at the margin the individual is indifferent the between consumption and investment, equations *(5)* and *(6)* set as equal the marginal benefits derived from the allocation of time that is equality between the aggregate gain from the last instant of leisure and the one from the last instant of studying and equality between the aggregate gain from the last instant of working and the one from the last instant of studying.

Equation *(7)* equalizes the marginal gain in utility from investing in physical capital to the gain from postponing the accumulation which is equal to the difference between the cost of the foregone consumption and the change in the shadow price of physical capital.

Equation *(8)* equalizes the marginal gain in utility from investing in human capital, in terms of both production of goods and new human capital, to the gain from postponing its accumulation which is equal to the difference between the cost of the foregone leisure and the change in the shadow price of human capital.

We now describe a solution imposing constant rates of growth.

Consider *(4)*, take logs and derivatives with respect to time, to obtain:

$$-\sigma\chi = (d\theta_{1t}/dt)/\theta_{1t} \tag{11}$$

Divide both sides of *(7)* by σ_{1t} and substitute from *(11)*; rearranging terms, we obtain:

$$(\pi + \sigma\chi)/\beta = AK_t^{(\beta-1)}(u_t H_t)^{(1-\beta)} H_t^{\gamma} \tag{12}$$

Now divide both sides of *(3)* by K_t to get:

$$\xi \equiv (dK_t/dt)/K_t = AK_t^{(\beta-1)}(u_t H_t)^{(1-\beta)}) H_t^{\gamma} - N_t c_t/K_t \tag{13}$$

Substitute from *(12)* to get:

$$\xi = (\pi + \sigma\chi)/\beta - N_t c_t/K_t \tag{14}$$

bringing all the constants on the LHS, taking again logs and derivatives, we obtain:

$$0 = \xi - \lambda - \chi$$

that is:

$$\xi = \lambda + \chi \tag{15}$$

In simple words the rate of growth of per capita consumption and per capita physical capital are equal.

Now, consider again *(12)*, take logs and derivatives, to get:

$$0 = (\beta - 1)\xi + (1 - \beta + \gamma)\upsilon$$

Now substituting from *(15)*, we obtain:

$$\upsilon = (\chi + \lambda)(1 - \beta)/(1 - \beta + \gamma) \tag{16}$$

Clearly, per capita consumption and per capita human capital growth rates differ only because of the externality.

Now take logs and derivatives in *(6)* and substitute from *(11)*, to get:

$$-\sigma\chi + \beta\xi + (1-\beta)\,\nu + \gamma\nu = \nu + (d\theta_{2t}/dt)/\theta_{2t}$$

Substituting from *(15)*, one obtains:

$$(d\theta_{2t}/dt)/\theta_{2t} = \chi\,(\beta - \sigma) + \lambda\beta + \nu\,(\gamma - \beta) \tag{17}$$

Finally from *(5)*, taking logs and derivatives, we get:

$$(d\theta_{2t}/dt)/\theta_{2t} = \lambda - \nu \tag{18}$$

Therefore we have to solve the following system:

$$\nu = (\chi + \lambda)\,(1 - \beta)\,/\,(1 - \beta + \gamma) \tag{16}$$

$$(d\theta_{2t}/dt)/\theta_{2t} = \chi\,(\beta - \sigma) + \lambda\beta + \nu\,(\gamma - \beta) \tag{17}$$

$$(d\theta_{2t}/dt)/\theta_{2t} = \lambda - \nu) \tag{18}$$

the three unknowns being $(d\theta_{2t}/dt)/\theta_{2t}$, υ, χ.

Substituting for υ from *(16)* in *(17)* and *(18)*, equalizing the two equations and rearranging their terms, we obtain:

$$\chi\,[1 - \beta + (\beta - \sigma)/(1 + \gamma - \beta)] = 0$$

it follows that:

$$\chi = 0 \tag{19}$$

$$\nu = \lambda\,(1 - \beta)/(1 + \gamma - \beta) \tag{20}$$

$$(d\theta_{2t}/dt)/\theta_{2t} = \lambda\gamma/(1 + \gamma - \beta) \tag{21}$$

Hence endogenous growth does not occur. From *(17)* the rate of growth of aggregate capital is equal to the exogenous rate of growth of population.

Since: $$\upsilon = \lambda(1 - \beta)/(1 + \gamma - \beta)$$

from eq. *(2)* we obtain:

$$\delta(1 - l_t - u_t) = \lambda(1 - \beta)/(1 - \beta + \gamma)$$

which represents the steady state optimal value of studying time s_t

(22) $$s_t^* = \lambda(1 - \beta)/(1 + \gamma - \beta)\,\delta$$

Insert *(6)* in *(8)*, to obtain:

$$(d\theta_{2t}/dt)/\theta_{2t} = \pi - u_t\delta\gamma/(1 - \beta) - \delta(1 - u_t - l_t)$$

Substituting from eq. *(21)* and *(22)* we obtain the steady state values for leisure and working time

(23) $$u_t^* = [(\pi - \lambda)(1 - \beta)]/\delta(1 + \gamma - \beta)$$

(24) $$1_t^* = 1 - \pi(1 - \beta)/\delta(1 + \gamma - \beta)$$

Apart from the externality γ, the interpretation of the optimal steady state values of the variables concerning the allocation of time is simple. Studying time is equal to λ/δ : the individual must study only to provide the newborns with the same per capita human capital stock and the higher δ (the maximum rate of feasible accumulation of human capital) the lower is the needed investment in human capital. Note also that, since the flow of human capital (studying time) is independent from the rate of time preference, the same holds for its steady state level, for any initial known value.

Working time is positively related to π : the higher the rate of time preference, the lower the steady state physical capital per unit of labor (via the modified golden rule). In other words, a higher π induces a substitution of working for physical capital in production, implying lower leisure *(24)* for any given human capital.

Finally we look at the two transversality conditions. Both equation *(9)* and *(10)* are verified as long as $\pi > \lambda$ (8).

In Lucas' model, optimisation with respect to leisure causes the system to grow at the exogenous rate of growth of population.

The equation that makes the point is the optimality condition on leisure *(5)*, which then implies eq. *(18)*.

3. - The Romer Model with Leisure

Allowing for leisure, the Romer model of 1986 can be formalized as follows.

First of all we define the aggregate production function in the presence of endogenous working time.

$$Y_t = [(1 - l_t)\, N_t]^{(1-\beta)}]\, K_t^{\beta}\, \kappa^{\eta}$$

dividing both sides by N_t, we get the per capita production function,

$$y_t = (1 - l_t)^{(1-\beta)}\, k_t^{\beta}\, \kappa^{\eta}$$

For the sake of simplicity, we work with stationary population (as in Romer) and furthermore we normalize it to one.

Hence the planner maximises:

$$\text{(25)} \qquad \underset{\{c_t,\, l_t\}_0^{\infty}}{\text{Max}} \int_0^{\infty} \frac{[c_t^{(l-\sigma)} + l_t^{(l-\sigma)}]}{1 - \sigma}\, e^{-\pi t}\, dt$$

s.t.

$$\text{(26)} \qquad dk / dt = (1 - l_t)^{(1-\beta)}\, \kappa_t^{\beta}\, \kappa^{\eta} - c_t$$

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(8) As a matter of fact, in equation *(9)* θ_{1t} is constant and K_t grows at the rate λ and therefore the transversality condition is verified if and only if $\pi > \lambda$. Analogously, in eq. *(10)* the sum of the rates of growth of H_t and θ_{2t} is equal to λ and again eq. *(10)* is verified if and only if $\pi > \lambda$.

As usual, small letters are for per capita values and κ^{η} is the technological externality, where κ is defined as aggregate knowledge.

Furthermore, equilibrium in the capital markets requires

$$\kappa = N_t\, k_t = K_t\,,$$

In our case:

$$\kappa_t = k_t$$

Let us write the standard current Hamiltonian value:

$$R = [c_t^{(1-\sigma)} + l_t^{(1-\sigma)}] \,/\, (1-\sigma) + \theta_t\, [(1-l_t)^{(1-\beta)}\, k_t^{\beta}\, \kappa^{\eta} - c_t]$$

The first order conditions for an interior maximum are:

(27) $$c_t^{-\sigma} = \theta_t$$

(28) $$l_t^{-\sigma} = \theta_t (1-\beta)(1-l_t)^{-\beta}\, k_t^{\beta+\eta}$$

(29) $$d\,\theta_t' / dt = \pi\,\theta_t - \theta_t (\beta+\eta)\, k_t^{(\beta+\eta-1)} (1-l_t)^{(1-\beta)}$$

(30) $$dk / dt = (1-l_t)^{(1-\beta)}\, k_t^{\beta}\, \kappa^{\eta} - c_t$$

(31) $$\lim_{t->\infty}\; e^{-\pi t}\, k_t\, \theta_t = 0$$

In order to find solutions with constant rates of growth proceed, as usual, as follows; take logs and derivatives of *(27)*, to get:

(32) $$[(d\,\theta_t / dt)] \,/\, \theta_t = -\,\sigma\,\chi$$

Now divide both sides of *(29)* by θ_t and substitute from *(32)*, to obtain:

(33) $$\pi + \sigma\,\chi = (1-l_t)^{(1-\beta)} (\beta + \eta)\, k_t^{(\beta+\eta-1)}$$

Now dividing both sides of *(30)* by k_t, one obtains the rate of growth of per capita capital:

$$\gamma_k = (1 - l_t)^{(1-\beta)} k_t^{(\beta+\eta-1)} - c_t / k_t$$

Substituting for $(1 - l_t)^{(1-\beta)} k_t^{(\beta+\eta-1)}$ from *(33)*, bringing all the constants on the LHS and taking logs and derivatives, one obtains

$$0 = - \chi + \gamma_k$$

that is:

$$\chi = \gamma_k \tag{34}$$

in simple words, the rate of growth of per capita consumption and per capita capital are equal.

From *(28)* taking logs and derivatives we get:

$$0 = [(d\,\theta_t / dt)] / \theta_t + (\beta + \eta)\,\gamma_k \tag{35}$$

$$[(d\,\theta_t / dt)] / \theta_t = - (\beta + \eta)\,\gamma_k$$

Therefore we have to solve the system of equations *(32)*, *(34)* and *(35)*.

$$[(d\,\theta_t / dt)] / \theta_t = - \sigma\,\chi \tag{32}$$

$$\chi = \gamma_k \tag{34}$$

$$[(d\,\theta_t / dt)] / \theta_t = - (\beta + \eta)\,\gamma_k \tag{35}$$

As long as $\sigma \neq \beta + \eta$, which is true since $\beta + \eta = 1$, χ and γ_k are equal to zero and therefore once again we are back to the neoclassical world (9).

(9) The same results hold for the market economy. In this case the maximization problem is the same as the one solved by the social planner, thanks to the production function, which is CRS, κ^η being considered as given by the individuals. The FOC for a maximum are all the same but eq. *(30)*. Individuals do not derive with respect to κ hence η does not appear on the r.h.s. as coefficient, but only as exponent. The following analysis proceeds on the same lines and the final results of zero per capita growth rates are not altered. The technological externalities obviously still play a role, affecting the steady state levels of per capita values, not their rates of growth.

Finally, looking at the transversality condition of eq. *(31)* we see that, since k_t and θ_t are constant, the condition is satisfied as long as $\pi > 0$.

4. - Why Does Leisure Lead us Back to Exogenous Growth?

We now intend to provide a simple interpretation of our mathematics. The key equation leading to exogenous growth is the optimality condition ruling the choice of leisure. Examining the FOC for a maximum in our version of the Romel model, we clearly have to equate over time the weighted marginal utilities of consumption and leisure. If there were endogenous growth, per capita consumption would grow unbounded and its marginal utility would tend to zero. As leisure must be constant in steady state, having an obvious upper limit, the growth of capital should increase leisure's opportunity cost (marginal benefit from working) in order to make up for the decrease in the marginal utility of consumption and keep the equality of marginal benefits *(28)*. Remembering that the growth of per capita capital is always equal to the growth of per capita consumption, θ's dynamics should follow the preferences (σ, eq. *(27)*) and the technology ($\beta + \eta$, eq. *(28)*). But there is no reason for preferences and technological parameters to be consistent in order to make up for the scarcity of time, which then causes exogenous growth.

Careful attention must be paid to the scarcity of resources. It is well known that, in the marginalistic tradition, decreasing returns originate from the existence of fixed production factors. This constraint is avoided by endogenous growth models thanks to the reproducibility of a number of factors sufficient to ensure constant returns. Such a reproducibility stems from the use of the resource "time", the fraction of which devoted to market activities is a production factor; for example, in Lucas, market activities, studying and working time, accumulate human and physical capital respectively.

All endogenous growth models, however, make two strong assumptions about the resource "time":

1) The reproduction of each factor occurs at constant return:

i.e., each instant of studying and working becomes steadily more and more productive;

2) The opportunity cost of the complex of "market activities" is zero, as leisure is neglected in the preferences; consequently, time is a free resource. The fixed fraction of time devoted to market activities does not affect the growth rates of the variables but only their levels; in other words, this fraction is a parameter of the model, exactly as the savings rate (the fraction of income saved) in the original Solow [11] growth model.

The explicit consideration of leisure gives the market activities a positive opportunity cost, thus turning time into an economically significant resource. Now it is clear that a new, unavoidable scarcity is at work; either leisure or market activities, once endogenously determined, cannot exceed the extent of the current period, whatever its length. Since leisure is a normal good, a positive steady growth of consumption generates an income effect which induces a reallocation of time from market activities to leisure. But, limiting the analysis to the steady state, leisure must be constant because of its upper bound, i.e. because of the scarcity of "time".

Thus, the scarcity of a relevant production factor reintroduces a sort of decreasing returns in this class of models: it follows that, as in the standard neoclassical model, despite the CRS technology, in equilibrium there is no room for balanced paths with per capita income growth.

5. - Conclusions

In this paper we have tried to show that a CRS technology on reproducible factors is not, by itself, a sufficient condition for endogenous growth. The literature has always relied on the CRS hypothesis and it has implicitly assumed that accumulation activity was not under choice. Actually the entire time optimally allocated, either by the planner or by the individuals, is dedicated to accumulating reproducible factors, which have CRS: the endogenous growth is therefore a necessary result. In other words, CRS is a sufficient condition if and only if the allocation of time is never affected by the rates of growth or by the levels of the variables.

In order to release such an implicit hypothesis, we have re-examined some representative endogenous growth models with the explicit introduction of leisure as a source of welfare for the individuals. This gives an opportunity cost to "market activities" (working + studying time), turning time into a production factor and making its scarcity relevant. The analysis leads back to exogenous growth, notwithstanding CRS on reproducible factors, confirming our argument.

A further development of our idea should address the behavior of the system without the hypothesis of constant growth rates. The structure of the model readily suggests the possibility of positive per capita growth, something that we showed to be unfeasible only at constant rates. The hypothesis of constant rates of growth seems to add a further constraint to the optimisation problem. As a matter of fact, in the neoclassical model it is shown that the steady state solution, investigated for analytical convenience, is stable and furthermore is the only one to satisfy all the optimality conditions: hence it does not take us away from the first best. On the other hand, the role of this hypothesis is not well understood in the endogenous growth literature, there being no complete stability analysis around an increasing growth path. Our presumption is that, once the traditional saddle paths around the steady state have been abandoned, there could be no more equivalence between the first best growth path and constant rates of growth. In such a case further investigation should give up the traditional exponential paths (10).

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